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# Large mixing and large hierarchy between neutrinos with Abelian flavor symmetries

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Abstract: The experimental data on atmospheric and solar neutrinos suggest that there is near-maximal mixing between $\nu_\mu$ and $\nu_\tau$ but that their masses are hierarchically separated. In models of Abelian horizontal symmetries, mixing of $\mathcal{O}(1)$ generically implies that the corresponding masses are of the same order of magnitude. We describe two new mechanisms by which a large hierarchy between strongly mixed neutrinos can be achieved in this framework. First, a *discrete* Abelian symmetry can give the desired result in three ways: mass enhancement, mixing enhancement and mass suppression. Second, *holomorphic zeros* can give mass suppression.

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## Contents





## 1. Introduction

The Super-Kamiokande collaboration recently announced evidence for neutrino masses [1]. Specifically, various measurements of the flux of atmospheric neutrinos can be explained by $\nu_\mu \to \nu_\tau$ oscillations with (for recent analyses, see [2, 3, 4])

$$\Delta m^2_{23} \sim 2 \times 10^{-3}\ eV^2\,, \qquad \sin^2 2\theta_{23} \sim 1\,. \tag{1.1}$$

The solar neutrino flux has been measured by various experiments. The data from the chlorine, GALLEX, SAGE and Super-Kamiokande experiments can be explained by $\nu_e \to \nu_x$ ($x = 2$ or 3) oscillations with one of the following three options (for a recent analysis, see [5]):

$$\begin{array}{ccc} & \Delta m^2_{1x}[eV^2] & \sin^2 2\theta_{1x} \\ \text{MSW(SMA)} & 5\times 10^{-6} & 6\times 10^{-3} \\ \text{MSW(LMA)} & 2\times 10^{-5} & 0.8 \\ \text{VO} & 8\times 10^{-11} & 0.8\,. \end{array} \tag{1.2}$$

Here MSW refers to matter-enhanced oscillations, VO refers to vacuum oscillations, and SMA (LMA) stand for small (large) mixing angle. Only central values are quoted for the various parameters. (We assume that there are no light sterile neutrinos. Otherwise a light $\nu_s$ could replace $\nu_\tau$ in (1.1) or $\nu_x$ in (1.2).)

With three light massive neutrinos, there are nine new flavor parameters in addition to the thirteen of the Standard Model: three neutrino masses, three mixing angles and three CP-violating phases. If the discrepancy between experiments and theory for both atmospheric and solar neutrinos is indeed explained by (1.1) and (1.2), then four of these new parameters have been measured. This information has strong impact on models that explain the observed smallness and hierarchy in flavor parameters. In this work, we focus on models of Abelian horizontal symmetries.

A crucial point concerning the combination of (1.1) and (1.2) is that the two mass-squared differences are widely separated:

$$10^{-7} \lesssim \frac{\Delta m^2_{1x}}{\Delta m^2_{23}} \lesssim 10^{-2} . \tag{1.3}$$

As a consequence of (1.3), there are only two different forms of neutrino mass matrices in the charged lepton mass basis that are consistent with both (1.1) and (1.2) [6]. One of these forms describes a situation where the parameters relevant to $\nu_\mu - \nu_\tau$ oscillations effectively reside in the $2 \times 2$ sub-matrix of the full $3 \times 3$ light neutrino mass matrix. The $\nu_\mu - \nu_\tau$ block is then of the form (we ignore CP violation):

$$\begin{aligned} M^{(2)} &= \frac{v^2}{M} \begin{pmatrix} C & B \\ B & A \end{pmatrix} , \\ A, B, C = \mathcal{O}(1) \, , &\qquad |AC - B^2| \ll B^2 \, . \end{aligned} \tag{1.4}$$

In addition, there is one form of the $3 \times 3$ matrix that gives large $s_{23} \equiv \sin\theta_{23}$ and cannot be effectively reduced to the $2 \times 2$ description of atmospheric neutrinos, that is [6]

$$\begin{aligned} M^{(3)} &= \frac{v^2}{M} \begin{pmatrix} 0 & B & A \\ B & 0 & 0 \\ A & 0 & 0 \end{pmatrix} , \\ A, B &= \mathcal{O}(1). \end{aligned} \tag{1.5}$$

In models of (supersymmetric) Abelian horizontal symmetries, large $\nu_\mu - \nu_\tau$ mixing is generically achieved with either [7]

$$\begin{aligned} M^{(2)} &= \frac{v^2}{M} \begin{pmatrix} C & B \\ B & A \end{pmatrix} , \\ A, B, C = \mathcal{O}(1), &\qquad AC - B^2 = \mathcal{O}(1) \, , \end{aligned} \tag{1.6}$$

corresponding to equal horizontal charges for the two lepton doublets, or with [8]

$$\begin{aligned} M^{(2)} &= \frac{v^2}{M} \begin{pmatrix} c & B \\ B & a \end{pmatrix} , \\ B = \mathcal{O}(1), &\qquad a, c \ll 1 \, , \end{aligned} \tag{1.7}$$

corresponding to opposite horizontal charges for the two lepton doublets. Thus, it is non-trivial to have in this framework large mixing between neutrinos whose masses are widely separated. It is the purpose of this work to describe the various ways by which mass matrices of the forms (1.4) or (1.5) can arise in models of Abelian horizontal symmetries and, in particular, to suggest two new methods for generating (1.4) or (1.5).

The structure of this paper is as follows. In section 2, we clarify some subtleties concerning the low-energy effective theory for neutrinos with an approximate flavor symmetry. In section 3, we review previously proposed mechanisms for inducing a large hierarchy. We then suggest two new mechanisms for producing a large hierarchy while maintaining large mixing. One, employing discrete symmetries, is described in section 4. The other, based on holomorphic zeros, is discussed in section 5. Finally, we present our conclusions in section 6.

## 2. Neutrino mass matrices with Abelian horizontal symmetries

We study Supersymmetric models with an Abelian horizontal symmetry $H$. For a simple $H$, we assume that there is a single small breaking parameter. Without loss of generality, we assume that the breaking parameter carries charge $-1$ under the horizontal symmetry. (By this we mean that the symmetry is broken spontaneously by a VEV of a scalar, Standard Model singlet field to which we attribute horizontal charge $-1$. The small parameter is then the ratio between this VEV and the scale where the information about the breaking is communicated to the observable sector [9].) If the horizontal symmetry is semi-simple, then we assume that each simple subgroup is broken by a single small parameter. Well below the $H$-breaking scale $\Lambda_H$, we have an effective theory with the following selection rules:

(a) Terms in the superpotential that carry charge $n \geq 0$ under $H$ are suppressed by $\mathcal{O}(\lambda^n)$, while those with $n < 0$ are forbidden (due to the holomorphy of the superpotential [10]).

(b) Terms in the Kähler potential that carry charge $n$ under $H$ are suppressed by $\mathcal{O}(\lambda^{|n|})$.

In the case of neutrinos, there is however a subtle point concerning the natural mass scale of the singlet neutrinos. The mass of these singlet neutrinos could in principle be lower than the $H$-breaking scale. For example, total lepton number may be a symmetry of the full theory that is broken only at a scale $\Lambda_L$ that is much lower than the $H$-breaking scale, $\Lambda_L \ll \Lambda_H$. Then, the low-energy effective theory below $\Lambda_H$ includes both the doublet ('left-handed') and singlet ('right-handed') neutrino fields. We will denote lepton-doublet fields by $L_i$ and singlet neutrino fields by $N_i$, with $i$ a flavor index. One can further integrate out the singlet neutrinos to obtain an effective theory for the left-handed neutrinos only, which is valid well below $\Lambda_L$. The selection rules cannot, however, be simply applied to this effective theory. In particular, terms in the superpotential that carry charge $n < 0$ under $H$ either vanish or are *enhanced* by $\mathcal{O}(\lambda^{-n})$. The latter possibility arises because the light neutrino mass matrix is given by

$$M_\nu^{\text{light}} = M_D(M_N)^{-1}M_D^T\,, \tag{2.1}$$

where $M_D$ is the Dirac mass matrix that couples the doublet and singlet neutrinos, while $M_N$ is the Majorana mass matrix for the singlet neutrinos. Then $M_\nu^{\text{light}}$ depends on negative powers of the heavy neutrino masses which are themselves suppressed by powers of $\lambda$. Therefore, in this type of models, one has in general to analyze the full neutrino mass matrix in order to estimate the flavor parameters in the light neutrino sector.[1]

Another possibility is that the scales $\Lambda_H$ and $\Lambda_L$ are the same. This will be the case if there is no lepton number symmetry in the full theory and if all Standard Model singlet neutrinos appear in vector representations of $H$. Then one can study the effective low energy theory with doublet neutrinos only, and apply the selection rules (a) and (b) given above. In particular, superpotential terms with negative $H$-charge should be set to zero. One can always find a full high energy theory (of the type described here, namely with no singlet fermion representations that are chiral under $H$) which yields the deduced flavor structure of the light neutrinos.

In the models we present below, whenever we analyze the mass matrix for both doublet and singlet neutrinos, we implicitly assume that the full high energy theory is of the first type described above ($\Lambda_L \ll \Lambda_H$), while when we analyze the effective theory for the doublet neutrinos only, we assume a full high energy theory of the second type ($\Lambda_L = \Lambda_H$).

## 3. Previously proposed mechanisms

### 3.1. Enhanced masses

In models with $\Lambda_L \ll \Lambda_H$, a neutrino mass could be enhanced beyond the naive expectation by the see-saw mechanism. To understand the mechanism in more detail, suppose that one of the entries in $M_\nu^{\text{light}}$, the effective $3 \times 3$ matrix for the light (left-handed) neutrinos, carries a certain horizontal charge $n$. Assume further that there exists a breaking parameter $\varepsilon$ with charge $m$ such that $p = n/m$ is a *positive integer*. (The $\varepsilon$ parameter could be either the only breaking parameter or one of several breaking parameters. Some or all of the other breaking parameters could have a sign opposite to that of $n$.) In the usual supersymmetric Froggatt-Nielsen mechanism for charged fermions, the entry in question cannot depend on $\varepsilon$ because of holomorphy. For neutrinos, however, this entry could get an *enhanced* contribution proportional to $\varepsilon^{-p}$.

[1] If all neutrino charges are positive, so that there are no holomorphic zeros in $M_D$ and in $M_N$, one can estimate the order of magnitude of the various light neutrino masses and mixings independently of the singlet neutrino charges [11, 7].



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Whether this happens depends on whether there are singlet neutrinos with masses suppressed by at least $\varepsilon^p$. This idea is presented and demonstrated by an explicit example in ref. [6].

We present a somewhat different model mainly for pedagogical reasons: it enables us to elucidate the crucial points of this mechanism and the subtleties discussed in section 2. Our model, however, cannot be made consistent with the detailed requirements of (1.2).

The horizontal symmetry is $U(1)_H$ and it is broken by a small parameter $\lambda$ to which we attribute charge $-1$. The neutrino fields carry the following $H$-charges:

$$L_1(y)\,, \quad L_2(-x)\,, \quad L_3(-x)\,, \quad N_1(a)\,, \quad N_2(b)\,, \quad N_3(c)\,, \tag{3.1}$$

with

$$y \geq x > 0\,, \qquad a \geq b \geq c \geq 0\,. \tag{3.2}$$

All entries in the $2-3$ block of $M_\nu^{\text{light}}$ carry charge $-2x$. Therefore, each of them could be either zero or enhanced by $\lambda^{-2x}$. Which of the two options is realized depends sensitively on the $N_i$-content of the model. Applying the selection rules to the $M_D$ and $M_N$ blocks of the full $6\times 6$ mass matrix, we get:[2]

$$M_D \sim \langle\phi_u\rangle \begin{pmatrix} \lambda^{a+y} & \lambda^{b+y} & \lambda^{c+y} \\ (\lambda^{a-x}) & (\lambda^{b-x}) & (\lambda^{c-x}) \\ (\lambda^{a-x}) & (\lambda^{b-x}) & (\lambda^{c-x}) \end{pmatrix}, \tag{3.3}$$

where the terms in parenthesis are non-zero only for non-negative exponents, and

$$M_N \sim \Lambda_L \begin{pmatrix} \lambda^{2a} & \lambda^{a+b} & \lambda^{a+c} \\ \lambda^{a+b} & \lambda^{2b} & \lambda^{b+c} \\ \lambda^{a+c} & \lambda^{b+c} & \lambda^{2c} \end{pmatrix}. \tag{3.4}$$

One can now distinguish between three interesting cases:

(i) $a < x$: Only one of the light neutrinos is massive with mass of order $\frac{\langle\phi_u\rangle^2}{\Lambda_L}\lambda^{2y}$, while the other two neutrinos are massless. Indeed, in this case none of the $N_i$ masses is suppressed by as much as $\lambda^{2x}$, so that the holomorphic zeros are maintained.

(ii) $x \leq b$: The three light neutrinos are massive with masses (relative to the scale $\frac{\langle\phi_u\rangle^2}{\Lambda_L}$) of order $\{\lambda^{-2x}, \lambda^{-2x}, \lambda^{2y}\}$. In this case two (or all three) of the $N_i$ have their masses below $\Lambda_L\lambda^{2x}$, thus allowing an enhancement of the two relevant light neutrino masses.

(iii) $b < x \leq a$: Two of the light neutrinos are massive with masses of order $\{\lambda^{-2x}, \lambda^{2y}\}$ and one neutrino is massless. Now only one of the $N_i$ has its mass below $\Lambda_L\lambda^{2x}$, and consequently the holomorphic zeros are lifted in such a way that only one of the light neutrinos has its mass enhanced by $\lambda^{-2x}$.

[2] We remind the reader that we only write the dependence of the various entries on the small breaking parameters and omit the unknown $\mathcal{O}(1)$ coefficients. The latter are arbitrary except that $M_\nu^{\text{light}}$ and $M_N$ are symmetric.



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In all three cases, the $\nu_\mu - \nu_\tau$ mixing is large, namely $s_{23} \sim 1$. Actually, while the $s^\nu_{12}$ and $s^\nu_{13}$ angles in the diagonalizing matrix of $M_\nu^{\rm light}$ may be different from one case to another, we generically expect those of the charged lepton sector to be the same in all cases, and consequently we get for the mixing angles:

$$s_{12} \sim \lambda^{x+y}\,, \qquad s_{13} \sim \lambda^{x+y}\,, \qquad s_{23} \sim 1\,. \tag{3.5}$$

Case (iii) above is of special interest to us, since together with the large $s_{23}$ it gives a strong hierarchy between the corresponding masses. This case therefore constitutes an example of the proposed mechanism: the mass of one combination of neutrinos with $H$-charges $-x$ is enhanced by the horizontal symmetry, while the other is not.

As mentioned above, the model of case (iii), while pedagogically useful, cannot satisfy all our phenomenological requirements. The light neutrino mass matrix, in the basis where the $2-3$ block is diagonal, is given by

$$M_\nu^{\rm light} \sim \frac{\langle \phi_u^2 \rangle}{\Lambda_L} \begin{pmatrix} \lambda^{2y} & 0 & \lambda^{x+y} \\ 0 & 0 & 0 \\ \lambda^{x+y} & 0 & \lambda^{-2x} \end{pmatrix}. \tag{3.6}$$

This structure cannot be made consistent with the VO or the MSW(LMA) solutions since the mixing angle is, at most, of order $(\Delta m^2_{1x}/\Delta m^2_{23})^{1/2} \ll 1$. It is also inconsistent with the MSW(SMA) solution since $\nu_e$ is heavier than $\nu_x$ (the light $\nu_\mu - \nu_\tau$ combination). With a different choice of charges, *e.g.* the one made in [6] ($x = -a = b = c$), one can get a matrix that is consistent with either of the large-angle solutions.

### 3.2. Neutrino masses from different sources

Different neutrino masses could come from different sources, so that the hierarchy is determined not only by the horizontal charges. A framework where this is the case is that of supersymmetry without $R$-parity. The Abelian horizontal symmetry could replace $R$-parity in suppressing the dangerous lepton-number violating couplings [12, 13, 14, 15, 16, 17, 18, 19]. If the $B$- and $\mu$-terms are not aligned, so that a single neutrino acquires a mass from mixing with neutralinos while the other two acquire masses at the loop level only, then the hierarchy between the heavier neutrino and the two light ones is too strong in general to accommodate both (1.1) and (1.2). But if $B$ and $\mu$ are aligned, all three neutrinos get loop-level masses. In a large class of such models, large mixing ($s_{23} \sim 1$) predicts a hierarchy $m_{\nu_2}/m_{\nu_3} \sim m_\tau^4/3m_b^4$. Recent studies of this framework were made in refs. [20, 21, 22, 23].

Another scenario in which different light neutrinos get their masses from different sources involves a single singlet neutrino. Then only one light neutrino acquires its mass at tree level, while masses for the other two are generated radiatively. This possibility was discussed in the context of the recent Super-Kamiokande data in ref. [24].

### 3.3. Accidental hierarchy

It may be that, as far as the small breaking parameters are concerned, it is model (1.6) which is realized in nature with the parameters of $O(1)$ accidentally giving a small determinant [25, 26, 27, 28]. If this is the case, then we are misled to think that (1.3) is related to an approximate horizontal symmetry. Indeed, the fundamental parameters are the neutrino masses and not the masses-squared. The ratio between the masses need only be $\mathcal{O}(0.1)$. This is not a very small number and so it is not impossible that it is accidentally (rather than parametrically) small. Rather plausible explicit examples for such a situation were given in refs. [26, 28].

## 4. Discrete symmetries

In all explicit examples that we construct in this (and in the next) section, we aim at the following order of magnitude estimates for the charged lepton masses:

$$m_e/m_\mu \sim \lambda^3\,, \qquad m_\mu/m_\tau \sim \lambda^2\,, \qquad m_\tau/\langle\phi_d\rangle \sim \lambda^3\,. \tag{4.1}$$

The last relation is appropriate for $\tan\beta \sim 1$ but all the models can be easily modified for larger values of $\tan\beta$. For the neutrinos, we require

$$s_{23} \sim 1\,, \qquad s_{13} \lesssim \lambda\,, \tag{4.2}$$

where the bound on $s_{13}$ comes from the combination of CHOOZ and Super-Kamiokande data [3, 4], and

$$\begin{array}{lcc} & \Delta m^2_{1x}/\Delta m^2_{23} & s_{12} \\ \text{MSW(SMA)} & \lambda^3-\lambda^4 & \lambda^2 \\ \text{MSW(LMA)} & \lambda^2-\lambda^4 & 1 \\ \text{VO} & \lambda^{10}-\lambda^{12} & 1 \end{array} \tag{4.3}$$

In our models of discrete symmetries, we take

$$H = Z_m \times Z_n \tag{4.4}$$

and breaking parameters

$$\varepsilon_m = \mathcal{O}(\lambda)\,, \qquad \varepsilon_n = \mathcal{O}(\lambda)\,. \tag{4.5}$$

The precise value of $n$ is not important except that it is large enough so that, for the fermion masses, the symmetry is effectively $Z_m \times U(1)$. On the other hand, the $Z_m$ symmetry is required to allow a mass hierarchy $m_2/m_3 \sim \lambda^m$ with $s_{23} \sim 1$. We will use then $m = 2$ for MSW solutions and $m = 5$ or 6 for VO models.



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### 4.1. Mass enhancement

A discrete symmetry can enhance one of the neutrino masses. Take, for example, an even $m$ with the following horizontal charges of the second and third lepton generations:

$$L_2(m/2-1,1)\,,\qquad L_3(m/2,0)\,,\qquad \bar\ell_2(m/2+1,4)\,,\qquad \bar\ell_3(m/2+1,2)\,. \tag{4.6}$$

Then, the $2\times 2$ mass matrices have the form

$$M_\nu^{(2)}\sim\frac{\langle\phi_u\rangle^2}{\Lambda_H}\begin{pmatrix}\lambda^m & \lambda^m\\ \lambda^m & 1\end{pmatrix}\,,\qquad M_{\ell^\pm}^{(2)}\sim\langle\phi_d\rangle\begin{pmatrix}\lambda^5 & \lambda^3\\ \lambda^5 & \lambda^3\end{pmatrix}\,. \tag{4.7}$$

When we rotate to the charged lepton mass basis, $M_\nu^{(2)}$ assumes the form (1.4). In particular, we get

$$\frac{m_{\nu_2}}{m_{\nu_3}}\sim\lambda^m\,,\qquad s_{23}\sim 1\,. \tag{4.8}$$

The crucial point to notice in (4.7) is that $(M_\nu^{(2)})_{33}$, which would have been $\mathcal{O}(\lambda^m)$ under a $U(1)$, is enhanced to $\mathcal{O}(1)$ under the discrete symmetry. The idea is then that horizontal charges that would lead to masses of the same order of magnitude with $H=U(1)$, can lead to hierarchical masses if in some of the mass terms the discrete nature of $H=Z_m$ comes into play. A large mixing angle could arise with either symmetry.

To give an example of the MSW(SMA) option, we take $m=2$ and the following charges:

$$L_1(0,3)\,,\quad L_2(0,1),\quad L_3(1,0)\,,\quad \bar\ell_1(0,5)\,,\quad \bar\ell_2(0,4)\,,\quad \bar\ell_3(0,2)\,. \tag{4.9}$$

Then, the mass matrices have the form

$$M_\nu\sim\frac{\langle\phi_u\rangle^2}{\Lambda_H}\begin{pmatrix}\lambda^6 & \lambda^4 & \lambda^4\\ \lambda^4 & \lambda^2 & \lambda^2\\ \lambda^4 & \lambda^2 & 1\end{pmatrix}\,,\qquad M_{\ell^\pm}\sim\langle\phi_d\rangle\begin{pmatrix}\lambda^8 & \lambda^7 & \lambda^5\\ \lambda^6 & \lambda^5 & \lambda^3\\ \lambda^6 & \lambda^5 & \lambda^3\end{pmatrix}\,, \tag{4.10}$$

yielding

$$\frac{\Delta m^2_{1x}}{\Delta m^2_{23}}\sim\lambda^4\,,\qquad s_{12}\sim\lambda^2\,,\qquad s_{23}\sim 1\,,\qquad s_{13}\sim\lambda^2\,. \tag{4.11}$$

A viable VO model is produced by $m=6$ and

$$L_1(1,2)\,,\quad L_2(2,1)\,,\quad L_3(3,0)\,,\quad \bar\ell_1(2,3)\,,\quad \bar\ell_2(4,4)\,,\quad \bar\ell_3(4,2)\,. \tag{4.12}$$

The mass matrices have the form

$$M_\nu\sim\frac{\langle\phi_u\rangle^2}{\Lambda_H}\begin{pmatrix}\lambda^6 & \lambda^6 & \lambda^6\\ \lambda^6 & \lambda^6 & \lambda^6\\ \lambda^6 & \lambda^6 & 1\end{pmatrix}\,,\qquad M_{\ell^\pm}\sim\langle\phi_d\rangle\begin{pmatrix}\lambda^8 & \lambda^{11} & \lambda^9\\ \lambda^8 & \lambda^5 & \lambda^3\\ \lambda^8 & \lambda^5 & \lambda^3\end{pmatrix}\,, \tag{4.13}$$

yielding

$$\frac{\Delta m^2_{1x}}{\Delta m^2_{23}}\sim\lambda^{12}\,,\qquad s_{12}\sim 1\,,\qquad s_{23}\sim 1\,,\qquad s_{13}\sim\lambda^6\,. \tag{4.14}$$

It is impossible to produce an MSW(LMA) model with $s_{13} \ll 1$. This fact is closely related to the $Z_2$ symmetry which, when the charges are chosen to give $s_{23} \sim 1$ and $s_{12} \sim 1$, always gives $s_{13} \sim 1$. More complicated models, *e.g.* models with a $Z_p \times Z_q \times Z_r$ symmetry, can accommodate the MSW(LMA) solution as well.

Even though the examples above all employ an even $m$, one can also construct models of mass enhancement with an odd $m$.

### 4.2. Mixing enhancement

A discrete symmetry can enhance a mixing angle. Take the horizontal charges of the second and third generation leptons to be

$$L_2(m-l,l)\,, \qquad L_3(0,0)\,, \qquad \bar{\ell}_2(l,5-l)\,, \qquad \bar{\ell}_3(l,3-l)\,. \tag{4.15}$$

The mass matrices then have, again, the form (4.7) and, consequently, (4.8) holds.

The mechanism that leads, however, to these results is different from that of the previous subsection. The crucial point here is that $(M^{(2)}_{\ell^\pm})_{23}$, which would have been $\mathcal{O}(\lambda^{m+3})$ under a $U(1)$, is enhanced to $\mathcal{O}(\lambda^3)$ under the discrete symmetry. The idea, then, is that horizontal charges that would lead to small off-diagonal terms with $H = U(1)$, can lead to unsuppressed off-diagonal terms with $H = Z_m$. A large hierarchy could occur with either symmetry.

It is, again, straightforward to construct explicit examples for the MSW(SMA) and the VO solutions, but it is impossible to construct an MSW(LMA) example with a suppressed $s_{13}$-mixing.

### 4.3. Mass suppression

A discrete symmetry can suppress neutrino masses. Consider models with singlet neutrinos at an intermediate scale $\Lambda_L$. Entries in $M_N$ could be enhanced, compared to a $U(1)$ model, by a discrete symmetry. (This mechanism is the same as the one discussed in section 4.1, except that now it operates on the singlet neutrinos.) This mass enhancement in the singlet neutrino sector translates, through the see-saw mechanism, to mass suppression in the light neutrino sector.

To give a concrete example (for MSW(SMA)), take $m = 2$ and the following charges:

$$\begin{aligned} &L_1(0,2),\ L_2(0,0)\,, \qquad L_3(0,0)\,, \qquad \bar{\ell}_1(0,6)\,, \qquad \bar{\ell}_2(0,5)\,, \qquad \bar{\ell}_3(0,3)\,, \\ &N_1(0,3), \qquad N_2(1,0)\,, \qquad N_3(1,0)\,. \end{aligned} \tag{4.16}$$

The mass matrices are of the form

$$M_D \sim \langle \phi_u \rangle \begin{pmatrix} \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda & \lambda \\ \lambda^3 & \lambda & \lambda \end{pmatrix}, \qquad M_N \sim \Lambda_L \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^4 & 1 & 1 \\ \lambda^4 & 1 & 1 \end{pmatrix}, \tag{4.17}$$

and $M_{\ell^\pm}$ is similar to that of eq. (4.10). The resulting parameters are the same as in eq. (4.11). Note, however, that the fact that the symmetry is discrete is irrelevant for both $M_D$ and $M_{\ell^\pm}$. It is only relevant for $M_N$, and the effect is an enhancement of two of its eigenvalues by $\mathcal{O}(\lambda^{-2})$. This, in turn, suppresses two of the light neutrino masses by $\mathcal{O}(\lambda^2)$, thus creating the required hierarchy.

Again, one can use this method to construct models that are consistent with the VO parameters but not (for $H = Z_2 \times U(1)$) with the MSW(LMA) parameters.

## 5. Suppressing a mass with holomorphic zeros

### 5.1. An effective-two-generation mechanism

We can use holomorphy to strongly suppress a neutrino mass ratio (compared to the naive estimate). The large mixing angle arises, in this case, from the charged lepton sector.

For simplicity, we start with a two generation model to demonstrate the proposed mechanism. The horizontal symmetry is $U(1) \times U(1)$, with the small breaking parameters both of $\mathcal{O}(\lambda)$. The charges are:

$$L_2(-1,1)\,, \qquad L_3(0,0)\,, \qquad \bar{\ell}_2(3,2), \qquad \bar{\ell}_3(3,0)\,. \tag{5.1}$$

Then, the mass matrices are of the form

$$M_\nu^{(2)} \sim \frac{\langle\phi_u\rangle^2}{\Lambda_H}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \qquad M_{\ell^\pm}^{(2)} \sim \langle\phi_d\rangle \begin{pmatrix} \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^3 \end{pmatrix}. \tag{5.2}$$

When we rotate to the charged lepton mass basis, $M_\nu^{(2)}$ assumes the form (1.4). We get

$$m_{\nu_2} = 0\,, \qquad m_{\nu_3} \sim \frac{\langle\phi_u\rangle^2}{\Lambda_H}, \qquad s_{23} \sim 1\,. \tag{5.3}$$

The zero mass of the light state is a generic feature of this mechanism in the two generation framework. It is, however, lifted (but remains suppressed) when the model is extended to include three generations.

The mechanism used here is similar in some aspects to the alignment mechanism of refs. [29, 30]. For both it is essential that we use a $U(1) \times U(1)$ symmetry rather than just a single $U(1)$. Assume that the symmetry is broken by small parameters of $\mathcal{O}(\lambda^m, \lambda^n)$. Suppose that a certain term carries charges $(H_1, H_2)$ under the horizontal symmetry. Then, we define the *effective* horizontal charge of this term, $\hat{H}$, by

$$\hat{H} = mH_1 + nH_2\,. \tag{5.4}$$

The basic idea is that a term with charge $\hat{H} \geq 0$ could still be forbidden by holomorphy if either of $H_1$ and $H_2$ is negative.

In alignment models, the charges of $Q_1$ and $Q_2$ give $\hat{H}(M_{12}) - \hat{H}(M_{22}) = 1$, so that naively $M_{12}/M_{22} \sim \lambda$. The charges of $\bar{u}_2$ and $\bar{d}_2$ are such that for $M^u_{12}$ both $H_1$ and $H_2$ are positive and the naive mass ratio holds, whereas for $M^d_{12}$ $H_1$ (or $H_2$) is negative and the ratio vanishes. In the neutrino models of this section, $\hat{H}(L_2) = \hat{H}(L_3)$. Naively then all entries in $M_\nu^{(2)}$ are of the same order of magnitude, but $(H_1, H_2)$ are such that holomorphy forbids all but $(M_\nu)_{33}$.

It is amusing to note that holomorphic zeros can lead to both a mixing angle that is much smaller than the corresponding mass ratio (alignment) and a mixing angle that is much larger than the corresponding mass ratio (the neutrino model of this section).

It is not trivial to extend the model to a three generation framework consistent with (1.2). In particular, we are only able to construct models that are consistent with large angle solutions to the solar neutrino problem (MSW(LMA) and VO in (1.2)). In these models, $\nu_e$ and $\nu_\mu$ form a pseudo-Dirac neutrino that is lighter than $\nu_\tau$. The mass scale of $\nu_\tau$ is then appropriate for (1.1), while the small mass splitting of the pseudo-Dirac neutrino explains (1.2). The effective low-energy theory is quite similar to the one described in section 6.2 of ref. [6], but the full theory (and, in particular, the mechanism to create the strong hierarchy) is very different.

To have a viable example of the MSW option, take for the first generation charges

$$L_1(1,0)\,, \qquad \bar{\ell}_1(3,4)\,. \tag{5.5}$$

Then, the mass matrices have the form

$$M_\nu \sim \frac{\langle\phi_u\rangle^2}{\Lambda_H}\begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 0 & 0 \\ \lambda & 0 & 1 \end{pmatrix}, \qquad M_{\ell^\pm} \sim \langle\phi_d\rangle \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^7 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^5 & \lambda^3 \end{pmatrix}, \tag{5.6}$$

yielding

$$\frac{\Delta m^2_{1x}}{\Delta m^2_{23}} \sim \lambda^3, \qquad \sin 2\theta_{12} = 1 - \mathcal{O}(\lambda^2)\,, \qquad s_{23} \sim 1\,, \qquad s_{13} \sim \lambda. \tag{5.7}$$

The $\mathcal{O}(\lambda^2)$ correction to $\sin 2\theta_{12} = 1$ should be quite large to satisfy the upper bound that applies to MSW(LMA), $\sin 2\theta_{12} < 0.9$. Note that if the only source for this correction were $(M_\nu)_{11}/(M_\nu)_{12} = \lambda$, then it would be accidentally suppressed, i.e., $\sin 2\theta_{12} = 1 - \lambda^2/8$. Therefore, the $\mathcal{O}(\lambda)$ correction to $s_{12}$ from the charged lepton mass matrix is important in making this class of models plausible candidates for the large angle option of the MSW mechanism.

An example that produces the VO option is the following:

$$L_1(2,2)\,, \qquad \bar{\ell}_1(6,-2)\,. \tag{5.8}$$

The mass matrices have the form

$$M_\nu \sim \frac{\langle\phi_u\rangle^2}{\Lambda_H}\begin{pmatrix} \lambda^8 & \lambda^4 & \lambda^4 \\ \lambda^4 & 0 & 0 \\ \lambda^4 & 0 & 1 \end{pmatrix}, \qquad M_{\ell^\pm} \sim \langle\phi_d\rangle \begin{pmatrix} \lambda^8 & \lambda^9 & \lambda^7 \\ 0 & \lambda^5 & \lambda^3 \\ 0 & \lambda^5 & \lambda^3 \end{pmatrix}, \tag{5.9}$$

yielding

$$\frac{\Delta m_{1x}^2}{\Delta m_{23}^2} \sim \lambda^{12}, \qquad \sin 2\theta_{12} = 1 - \mathcal{O}(\lambda^8)\,, \qquad s_{23} \sim 1\,, \qquad s_{13} \sim \lambda^4. \tag{5.10}$$

### 5.2. A three generation mechanism

Holomorphic zeros can easily reproduce the interesting three generation mass matrix (1.5). In this case one combination of $\nu_\tau$ and $\nu_\mu$ is the lightest neutrino while the orthogonal combination combines with $\nu_e$ to form a pseudo-Dirac neutrino.

Actually, an explicit model is easy to present. Just take the model of section 3.1 but assume that the full high energy theory has only singlet neutrinos in vector representations of $H$, namely, there are no singlet neutrinos at an intermediate scale (the $\Lambda_L = \Lambda_H$ case). Then, as explained in section 2, the selection rules can be applied directly to $M_\nu^{\rm light}$. The charges of $L_i$ of eq. (3.1),

$$L_1(y)\,, \quad L_2(-x)\,, \quad L_3(-x)\,, \quad (y \geq x > 0)\,, \tag{5.11}$$

lead to

$$M_\nu^{\rm light} \sim \frac{\langle\phi_u\rangle^2}{\Lambda_H}\begin{pmatrix} \lambda^{2y} & \lambda^{y-x} & \lambda^{y-x} \\ \lambda^{y-x} & 0 & 0 \\ \lambda^{y-x} & 0 & 0 \end{pmatrix}, \tag{5.12}$$

which is precisely the structure (1.5). Holomorphy here makes the whole $2-3$ block of $M_\nu^{\rm light}$ vanish. The matrix is then of rank 2, so that one neutrino, $-s_{23}\nu_\mu + c_{23}\nu_\tau$ (where $s_{23} = \mathcal{O}(1)$), is rendered massless. For the two components of the pseudo-Dirac neutrino, which we denote by $\nu_1$ and $\nu_x$, we have

$$m_{1,x} \sim \frac{\langle\phi_u\rangle^2}{\Lambda_L}\lambda^{y-x}\,, \qquad \frac{\Delta m_{1x}^2}{m_{1,x}^2} \sim \lambda^{y+x}\,, \qquad \sin 2\theta_{1x} = 1 - \mathcal{O}(\lambda^{2(y+x)})\,. \tag{5.13}$$

Since the scale of $m_x$ is set by (1.1) and the mass splitting $\Delta m_{1x}^2$ is set by (1.2), the ratio between them should be small and, consequently, so is the deviation of $\sin 2\theta_{1x}$ from unity. Therefore, this scenario can only fit the vacuum oscillation solution of the solar neutrino problem [6]. It requires $y + x \sim 10$.

## 6. Conclusions

The neutrino flavor parameters that seem to emerge from the observations of atmospheric and solar neutrinos are not easily accommodated in flavor models that explain the smallness and hierarchy in the charged fermion parameters. In particular, many of these models relate large mixing in the neutrino sector to non-hierarchical masses, while the most straightforward explanation of the experimental data requires $\sin\theta_{23} \sim 1$ and $m_{\nu_2} \ll m_{\nu_3}$.



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In this work, we investigated the implications of the neutrino parameters for flavor models with the following features: (a) Three light neutrinos; (b) Supersymmetry; (c) Abelian horizontal symmetry broken by a single parameter.

Within this framework, several mechanisms for obtaining large mixing together with a large hierarchy were suggested:

(i) Accidental hierarchy.

(ii) $R_p$ violation: different neutrino generations acquire their masses from different sources.

(iii) See-saw mass enhancement: a light neutrino mass is enhanced because it is acquired through a see-saw mechanism, and the corresponding singlet neutrino mass is suppressed.

Here we suggested several new mechanisms:

(iv) Mass enhancement from discrete symmetries: a light neutrino mass could be larger for a $Z_n$ symmetry than its would-be value for a $U(1)$ symmetry.

(v) Mixing enhancement from discrete symmetries: a mixing angle could be larger for a $Z_n$ symmetry than its would-be value for a $U(1)$.

(vi) Mass suppression from discrete symmetries: a singlet-neutrino mass is enhanced by a discrete symmetry, thus suppressing a light neutrino mass through the see-saw mechanism.

(vii) Holomorphic zeros: a neutrino mass is suppressed by holomorphic zeros in the mass matrix.

Outside our framework of supersymmetric Abelian horizontal symmetries there are, of course, many other proposals in the literature (some related to various symmetries and some which are simply ansatze for mass matrices) for achieving a large hierarchy together with large mixing [31]–[48]. In particular, SO(10) grand-unification, which in fact motivated the see-saw mechanism, generically gives maximal mixing between $\nu_\mu$ and $\nu_\tau$ [49].

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